Peak Friction Angle of Undisturbed Sands using DMT

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ABSTRACT: A database of effective stress friction of sands is compiled from six series of undisturbed samples of clean sands acquired using special field drilling methods, primarily one-dimensional freezing technologies or special piston tube samplers. At these sites, flat plate dilatometer tests (DMT) have also been performed. Once mounted in the laboratory triaxial apparatus, recovered specimens are then subjected to triaxial compression testing (CIUC, CAUC, CK_0UC) to measure their respective peak friction angles (ϕ'). Triaxial data are shown to conservatively match with available DMT interpretations using the horizontal stress index (K_D) developed by Marchetti (1985, 1997), thus validating the methodology with measured values from laboratory undisturbed tests. Pressuremeter tests (SBPMT) at four sand sites independently substantiate the ϕ' values and complement the above findings. Finally, case studies involving DMTs at two well-documented sand sites in Texas and Ireland are used for additional corroboration.

1 INTRODUCTION

1.1 *Effective friction angle*

The effective stress friction angle of sands (ϕ') is one of the most sought parameters in geotechnical analysis and design as it controls strength for foundation bearing capacity, axial pile response, and retaining walls, as well as mandatory input for finite element and numerical simulations. In the study reported herein, laboratory triaxial results from six sands and field pressuremeter measurements from four sands are reviewed within the context of DMT data and associated interpretation methods of ϕ'.

1.2 *Wedge penetration theory*

Marchetti (1985) utilized a solution from wedge penetration theory (Durgunoglu & Mitchell 1975) to develop a chart expressing normalized cone penetration resistance (q_c/σ_{vo}') in terms of effective friction angle (ϕ') and lateral stress coefficient (K_0) for clean sands. Campanella & Robertson (1991) extended this formula to horizontal stress index (K_D) by noting the average trend: (q_c/σ_{vo}') ≈ 33 K_D. Figure 1 shows the resulting interrelationship of ϕ' in terms of K_D and K_0. The individual lines for each specific value of ϕ' are clipped with a lower bound for K_0 established by the Rankine active stress coefficient: $K_A = (1-\sin\phi')/(1+\sin\phi')$; and upper bound given by the passive condition: $K_P = (1+\sin\phi')/(1-\sin\phi')$. Details on these derivations are discussed by Marchetti (1997) and Marchetti et al. (2006).

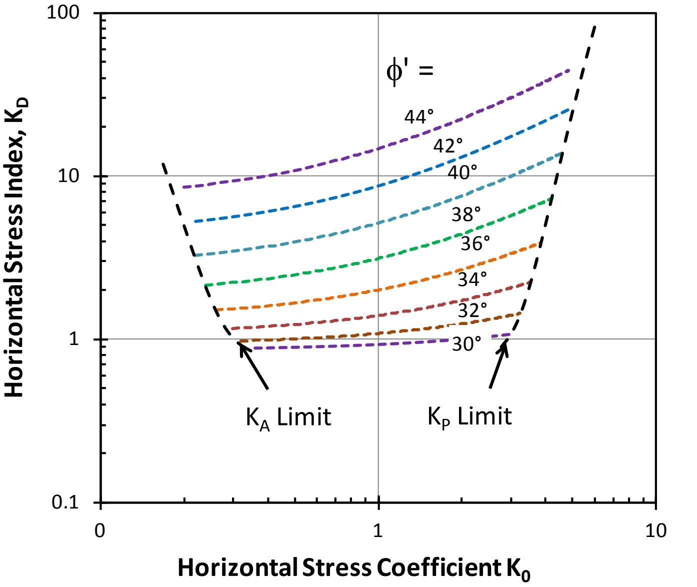


Fig.1. Wedge penetration theory for ϕ' in terms of K_D and K_0 (after Marchetti 1985 and Campanella & Robertson 1991)

The trends in Figure 1 can be approximately expressed by:

$$\phi' \approx 37.3° \cdot \left(\frac{K_D - 0.8}{K_0 + 0.8} \right)^{0.082} \quad (1)$$

with the appropriate bounds for K_0 given by K_A and K_P, as noted previously.

A series of direct relationships between ϕ' and K_D were produced by Marchetti (1997) by assigning three specific K_0 conditions: (a) K_{0NC} = 1 - sinϕ'; (b) K_0 = 1; and (c) a passive case condition taken with K_0 = sqrt (K_P). The resulting curves are shown in Figure 2 and these can be approximated by the following expressions:

(a) $$\phi' = 28.2° + \frac{(K_D - 0.5)}{0.074 + 0.063 \cdot (K_D - 0.5)^{0.92}} \quad (2)$$

(b) $$\phi' = 27.5° + \frac{(K_D - 0.5)}{0.080 + 0.063 \cdot (K_D - 0.5)^{0.94}} \quad (3)$$

(c) $$\phi' = 26.8° + \frac{(K_D - 0.5)}{0.10 + 0.062 \cdot (K_D - 0.5)^{0.95}} \quad (4)$$

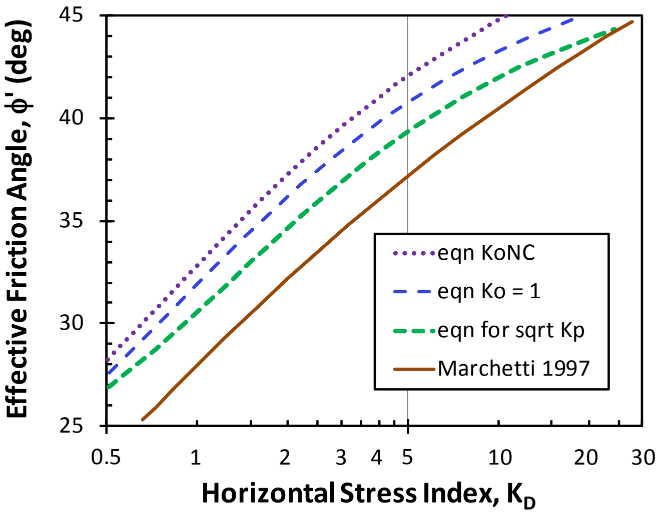


Fig.2. Sand friction angle ϕ' in terms of K_D for different lateral stress states (after Marchetti 1997)

Marchetti (1997) compared these curves with available calibration chamber results from DMTs in sands and concluded a slight overprediction in ϕ' at the initial portions of the curves, and therefore recommended a lower bound curve (also shown in Figure 2) given by:

$$\phi' = 28° + 14.6 \cdot \log(K_D) - 2.1 \cdot [\log(K_D)]^2 \quad (5)$$

2 DATABASE

2.1 *Undisturbed sands*

Direct measurements on the effective stress friction angle of sands can be obtained via undisturbed samples subjected to laboratory triaxial compression tests. While for many years unfeasible because of the difficulties in obtaining "undisturbed samples", the approach is now possible with the advent of special expensive frozen samples, as well as new gel samplers, Mazier tubes, and sonic sampling devices. Results from these lab reference tests can be used to check the lower bound DMT solution offered by equation (5).

A total of 6 sands that were sampled and tested under both laboratory triaxial compression (TC) tests and field DMT soundings are listed in Table 1. Types of laboratory TC series included isotropically-consolidated undrained (CIUC) and drained (CIDC), as well as anisotropically-consolidated tests (CAUC, CADC), prior to shearing.

Table 1. List of undisturbed sands tested by DMT, SBPMT and lab triaxial compression

Sand site	*Origin*	*D_{50} (mm)*	*Source for ϕ' (°)*	*References*
Holmen, Norway	primarily fluvial	0.3 to 0.7	CAUC, CADC, SBPMT	Lunne et al. 2003 *see note (a)
Massey, BC	alluvial	0.2	CIUC, CAUC	Cruz 2009 *see note (b)
Po River, Italy	alluvial	0.2 to 0.4	SBPMT	Ghionna et al. 1995
Kidd 2, BC	alluvial	0.17	CIUC, CAUC	Cruz 2009 see note (b)
Kowloon, China	hydraulic fill	0.72	CIUC, SBPTM	Lee et al. 1999
Blessington, Ireland	lacustrine delta	0.1 to 0.15	CIDC	Doherty et al. 2012 see note (c)
McDonald's Farm, BC	natural alluvial	0.12 to 0.5	CIDC	Robertson 1982

Notes:
(a) SBPMT data from Lacasse et al. (1990)
(b) triaxial data from Wride & Robertson (1998)
(c) additional information from Tolooiyan and Gavin (2011)

Using the triaxial tests as the reference ϕ' for the available sands, Figure 3 compares the various K_D expressions and confirms the lower bound is reasonable for assessing sand strength from the DMT.

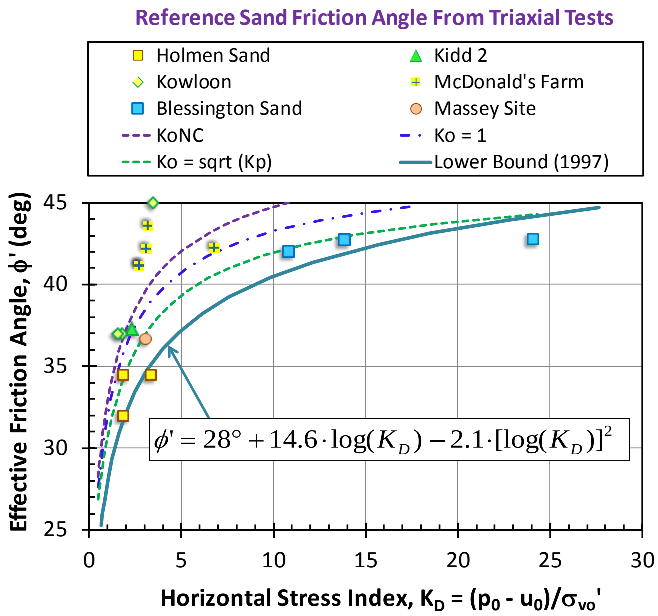
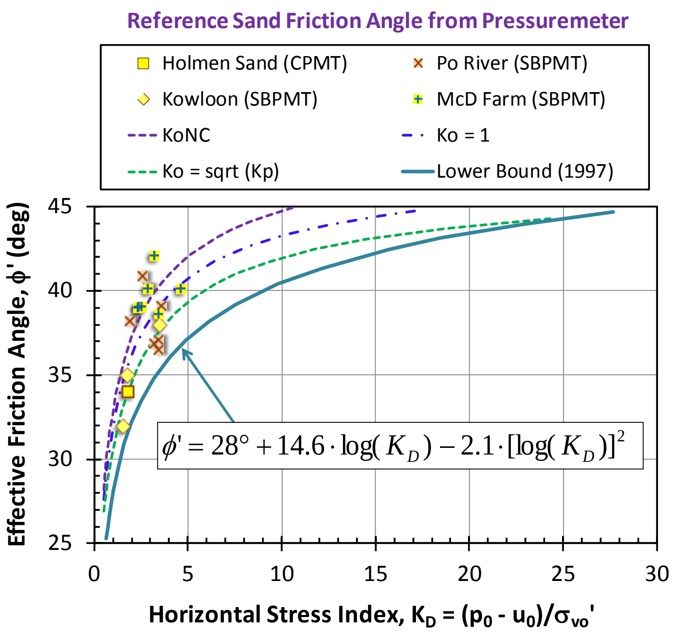


Fig. 3. Sand friction angle ϕ' from triaxial compression tests compared with K_D expressions by Marchetti (1997)

Fig.4. Sand friction angle ϕ' from pressuremeter tests



compared with K_D expressions given by Marchetti (1997)

2.2 *Pressuremeter testing*

Independent assessments on the effective friction angle of sands are afforded via data from self-boring pressuremeter tests (SBPMT), as detailed by Ghionna et al. (1995) for the Po River site in Italy. In addition, data at three of the previously noted sand sites were also subjected to SBPMT (Holmen, Kowloon, and McDonald's Farm) and these values have also been used to compare with the DMT-interpreted values. These 4 sand sites were subjected to both SBPMT and DMT, as listed in Table 1.

In Figure 4, a comparison of the reference ϕ' from pressuremeter tests and the K_D expressions for DMT are presented, again confirming the lower bound offered by equation (5).

3 APPLICATIONS

3.1 Texas A&M Sand Site

Established over three decades ago, the national geotechnical experimentation site (NGES) in sands at Texas A&M University has served for a wide variety of in-situ testing, geophysics, and full-scale construction projects involving many researchers, organizations, and governmental agencies (Briaud 2000). The site is underlain by fluvial and flood plain sedimentary deposits of Pleistocene age consisting of clean to silty to clayey sands within the upper 10 to 12 m. Groundwater generally lies about 5 m deep. The uppermost sand layer has a mean grain size D_{50} = 0.2 mm.

The measured profiles of cone tip resistance (q_t) and friction ratio ($FR = 100 \cdot f_s/q_t$) from a representative cone penetration test (CPT 06) performed at the site, together with the measured p_0 and p_1 readings from a nearby dilatometer sounding (DMT-3) are presented in Figure 5 (data from Gibbens & Briaud 1994).

For the CPT, a calibrated relationship has been developed for the assessment of ϕ' in sands from an elite database of 17 sands that were sampled using special and expensive high-quality methods (i.e., freezing) and, after thawing, trimmed specimens were subjected to consolidated triaxial compression testing in the laboratory (Mayne 2006; 2014). The derived expression for evaluating the peak ϕ' from the CPT is given by:

$$\phi' = 17.6° + 11.0 \cdot \log (q_{t1}) \qquad (6)$$

where $q_{t1} = (q_t/\sigma_{atm})/(\sigma_{vo}'/\sigma_{atm})^{0.5}$ = stress-normalized cone tip resistance (Jamiolkowski et al. 2001).

A comparison of the derived ϕ' profiles from both CPT and DMT results is shown in Figure 6. Over the depths investigated from 0 to 8 m, statistical analyses indicated the mean value (and ± one standard deviation) from the CPT gave ϕ' = 38.8° ± 1.6° and corresponding mean value from DMT gave ϕ' = 38.5° ± 1.4°, indicating quite excellent agreement amongst these two in-situ test relationships.

Another source for obtaining benchmark values of ϕ' is afforded by backcalculation from limit plasticity solutions using the "bearing capacity" measured in foundation load tests. Full scale load tests on large

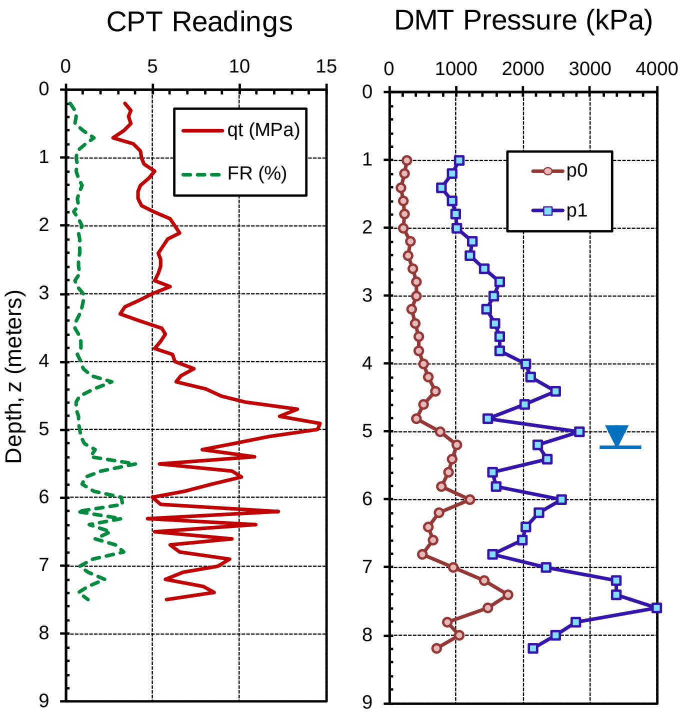


Fig. 5. Results from CPT and DMT soundings at Texas A&M sand site (data from Gibbens & Briaud 1994)

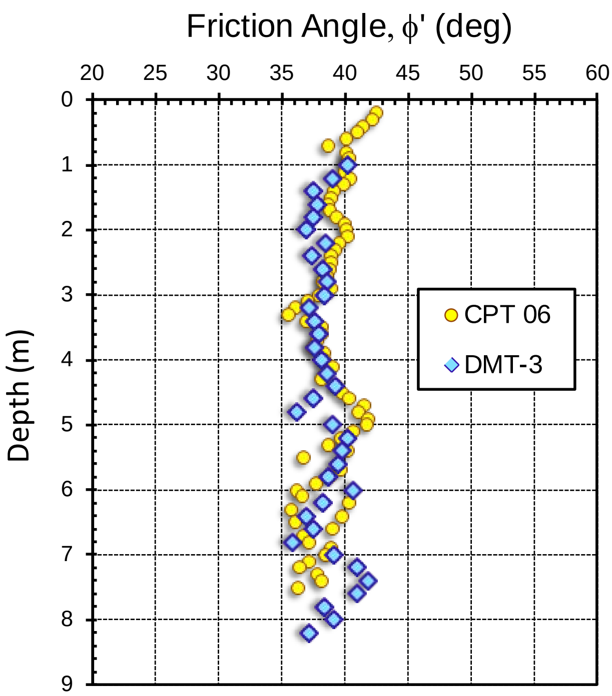


Fig. 6. Comparison of ϕ' profiles interpreted from CPT and DMT at Texas A&M sand site

spread footings are reported at this site by Briaud & Gibbens (1999). If an interpreted "capacity" is based on the s/B = 10% criterion, then the limit plasticity solution of Vesic (1975) gives a backfigured range of friction angles ϕ' = 40° to 42° for three large spread footings (one with B = 2.5 m and two with B = 3.0 m) at the Texas A&M site. See Mayne, Uzielli, & Illingworth (2012) for details on these calculations. The corresponding backfigured ϕ' would be appropriate in the depth ranges of 0.76 m (embedment depth) to about 3.76 m (approx. one B deep) and consistent with the DMT and CPT interpretations shown in Figure 6.

3.2 *Blessington sand site, Ireland*

The University College Dublin (UCD) has established an experimental test site for pile research in dense overconsolidated sands in eastern Ireland. Details are reported by Tolooiyan & Gavin (2011) and Doherty et al. (2012). These glacially-derived dense fine sands have an in-situ relative density around 100% and mean particle size: 0.10 < D_{50} (mm) < 0.15 mm. Sand mineralogy is predominantly quartz with calcite, feldspar, mica, and kaolinite. In-situ testing has included a series of four DMT soundings, as presented in Figure 7. Groundwater lies about 13 m deep.

Samples of the sand were procured by continuous sonic drilling for the laboratory test program, including triaxial compression testing for ϕ_p' evaluations and one-dimensional consolidation tests to define the yield stress (σ_y'). For the latter, per Casagrande criterion, the interpreted values of yield stress range from 320 kPa < σ_y' < 780 kPa in the upper 10 m.

The interpreted peak friction angles from the triaxial series and four DMTs are presented in Figure 8. The corresponding depths of the triaxial data are plotted at their appropriate value, as determined at the applied effective confining stresses (σ_c') divided by the unit weight of the sand (γ_t = 20 kN/m^3). As seen in Figure 8, the comparision between the DMT-interpreted ϕ' and lab triaxial values show very good to excellent agreement for these dense and overconsolidated sands.

4 CONCLUSIONS

A lower bound solution from Marchetti (1997) that relates the effective stress friction angle of sands (ϕ') at peak strength to the DMT horizontal stress index, K_D, is shown to provide reasonable results when compared to triaxial compression test data on undisturbed sand samples taken at 6 sites. Additional independent ϕ' evaluations from self-boring pressuremeter tests at 4 sand sites also indicate similar confidence in the lower bound relationship.

Case study results from two well-documented sand sites in Texas and Ireland showed excellent corroboration between DMT-interpreted profiles of

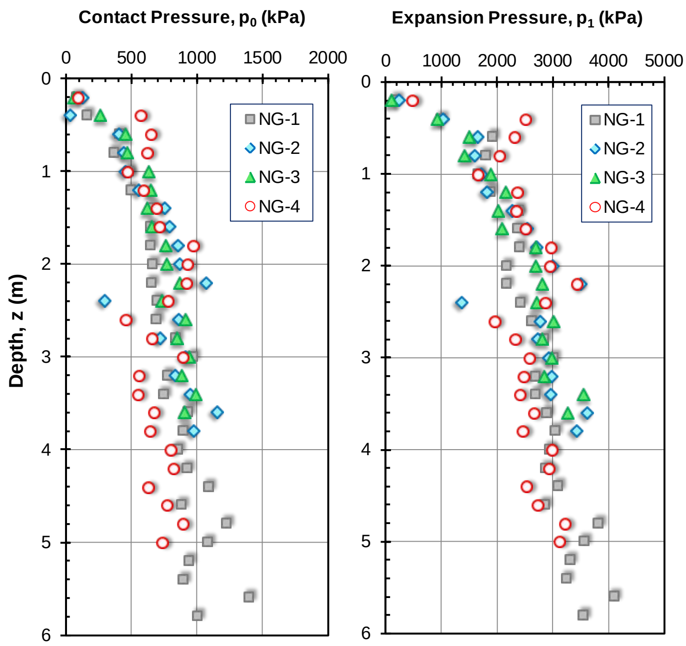


Fig. 7. Results of DMT soundings at Blessington sand site (data from Doherty et al. 2012)

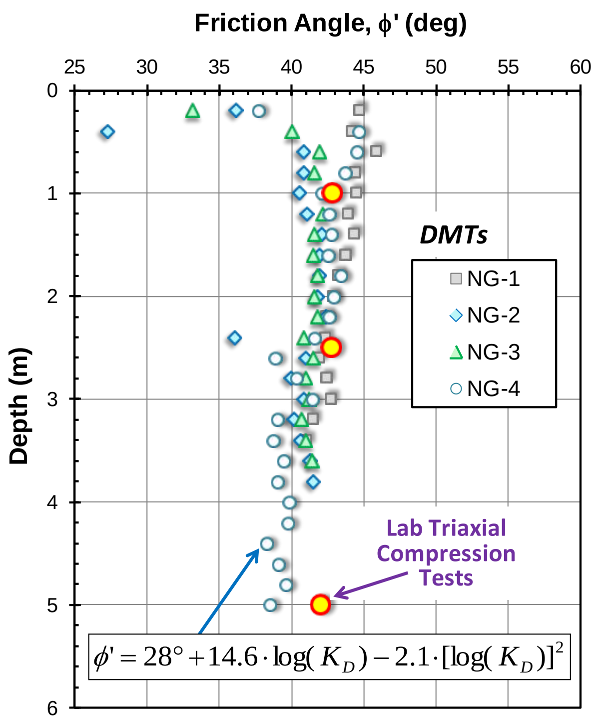


Fig. 8. Comparison of DMT-interpreted friction angles and reference triaxial values at Blessington sand site (data from Tolooiyan & Gavin 2011)

peak ϕ' values and reference benchmark values, including CPTs and triaxial tests. Furthermore, back-calculated ϕ' evaluations using limit plasticity theory and bearing capacities from large footing load tests at the Texas site give similar and comparable magnitudes of ϕ' to those in the DMT profiles.

5 ACKNOWLEDGMENTS

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